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ABSTRACT

The uses of verbal rules in solving arithmetic problems are presented. Thirty-six problems were used in individual testing of five students who were enrolled in a remedial mathematics course at a community college. Two or three weeks prior to the individual exams, the students had been tested in their class on the material. The study indicated that verbal rules provide a plan for action in verbal-retrievable form, but students must add a non-trivial amount of knowledge to make the rules effective. Five principles developed to explain the status and use of verbal rules in problem solving are presented and discussed in the context of the study. It is suggested that the next area of study be the process of transition from rules to procedures that occurs within the student's mind. (MP)

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A Study of Signed-number Arithmetic

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The Use of Verbal Rules in Problem Solving: A Study of Signed-number Arithmetic

Seth Chaiklin and James G. Greeno

Constructing and implementing a plan is one approach to solving problems. Another is to acquire a plan from another person or a book. In this case, these externally or authoritatively acquired plans can be called verbal rules. In this paper, we want to develop the idea of a verbal rule, and present some results about the properties of verbal rules and their role in problem solving. There are many kinds of verbal statements that might arguably be called verbal rules. We will focus on verbal rules that are stated in an imperative form: Do this to accomplish that. More precisely, we will be concerned with verbal rules that can be characterized as a description of actions, mental or physical, to be performed in order to accomplish a goal.

There are many motor and cognitive skills for which verbal rules can be formulated and used felicitously. For example, a golfer is advised to keep her left elbow straight and keep her eye on the ball. A manual instructs how to overhaul a bicycle. A student of grammar is told to "Put i before e, except after c," and a calculus student is instructed that to differentiate a monomial, "Multiply the coefficient by the power of the variable, and decrease the power by one."

Psychological properties of verbal rules. Theoretically, a verbal rule has at least three psychological functions. First, it is a plan for performance. In practice, verbal rules tend to be high-level plans; therefore they are incompletely or generally specified. Also, they

usually contain minimal or no indication of when that plan should be applied--that is, the application conditions are not always given. Second, verbal rules are in a linguistic form, thus they are easy to communicate and memorize. This linguistic form permits the communication of general or abstract principles in a way that is not possible with procedural knowledge which must be communicated via specific examples. Third, in addition to being linguistic, verbal rules are usually stated concisely, which contributes as well to their facile communication. The concise form also makes a verbal rule easier to memorize and retrieve. By way of example, the rule "i before e, except after c" is abstract in that it does not specify when or where this rule should be used; that it is easy to communicate, memorize and retrieve is probably verified by the reader's experience with this rule.

Information contained in a verbal rule. A verbal rule must contain descriptions of actions. It may also contain descriptions of application conditions, an implicit temporal sequence for carrying out the individual descriptions in the verbal rule, and a marking of conditions from actions by adverbial conjunctions. Consider this example of a verbal rule taken from the domain of signed-number arithmetic: When adding unlike-signed numbers, take the difference and keep the sign of the larger. The first part of this description is the application condition. It specifies when this verbal rule should be used. If application conditions are not stated explicitly, then clearly a person must formulate some. It happens that verbal rules for signed-number problems have explicit application conditions. This may be a consequence of there being more than one verbal rule in this domain

of arithmetic. Not all verbal rules contain explicit application conditions. Notice that the examples given in the second paragraph did not provide explicit descriptions of application conditions. However, the context in which a description is given may provide adequate information from which a person could infer a set of conditions that determine when it is (and is not) appropriate to apply that description. Similarly, abstract or partial specifications may be supplemented with additional application conditions to be satisfied. In the case of the signed-number rules, a person may specify what constitutes "adding" and "unlike-signed numbers".

The second part of this signed-number verbal rule is the action prescriptions. Action prescriptions tend to be a partial specification of the actions that should be performed if the application conditions are satisfied. In this example, the instruction, "Take the difference," carries an implicit requirement that the absolute value of the numbers be used. Further specification of action prescriptions is part of the process of semantically interpreting a verbal rule.

The temporal sequence for applying this signed-number verbal rule is implicit in the linear order in which it is stated. A person must first check for adding unlike signs, then take the difference, then give the sign of the larger. The separation of the conditions from the actions is marked by the adverbial conjunction "when".

Five principles have been derived about the status and use of verbal rules in problem solving. These principles are a start toward theories of the role of verbal statements in performance and of the

process of converting statements into actions. It also contributes to a theory about the use of plans in problem solving. To develop these principles, we examined the use of verbal rules in a domain of junior-high school arithmetic: addition and subtraction of positive and negative integers. This is a domain in which verbal rules are used significantly in instruction, and seem to be important in the performance of some students. In fact, the students and teachers even call the verbal rules "signed-number rules". In the remaining pages, the task domain and the data collection method will be described, and the five principles presented and discussed. The issues we will consider are what is needed to convert a verbal rule into an operational form, the conditions under which it is used, some features of mental procedures that are derived from verbal rules, and the role of parsing knowledge in the selection and execution of verbal rules. We will close with some conclusions about the role of verbal rules in performance and a speculation about a possible extension.

Method

Task domain. Thirty-six arithmetic problems--eighteen addition and eighteen subtraction--were used as the problem set. They were constructed by manipulating three structural features: order of magnitude for the two numbers (larger first or second), sign on the first number (+, -, or no sign), and sign on the second number. All combinations were used which resulted in 18 (2 x 3 x 3) possibilities. This generation procedure was used for both the addition problems and the subtraction problems. Numbers were selected randomly save for

observing the relative magnitude constraint. Examples include: -37 + 23" and "6 - -18".

Subjects. Five students who were enrolled in a remedial arithmetic course at a local community college were interviewed. These students had been instructed, as part of their regular coursework, to solve signed-number problems by applying a set of three verbal rules. Two or three weeks prior to the interviews, the students had taken an in-class exam on this material; in principle, they should have been able to solve signed-number problems competently at the time of the interviews.

Procedure. Each student was tested individually in a private room. They were given the problem set typed on two sheets of paper. They were also given scratch paper and told that they were free to use it in their computations. The students were instructed to read aloud each problem before starting to solve, to describe orally both what they were thinking while solving each problem and how they were solving it, and that they should use the methods they normally use. Students were instructed to write their answer next to each problem. No time limit on solution attempts was imposed. No feedback was given intentionally. Some students were occasionally reminded to think aloud. After solving the problem set, students were asked to describe any rules they might have used to solve these problems. They were also questioned informally about how they were solving particular problems, and about any difficulties they had. The entire interview was tape-recorded and transcribed.

(1) Verbal rules must be elaborated for successful operation. Verbal rules do not specify all the procedures needed for correctly solving a problem. In order to develop a theory of the knowledge needed to make verbal rules operational and to examine the status of verbal rules in a problem solving system, we constructed an operating production system model, written in ACTP (Anderson, 1976; Greeno, 1978). This model was based on a student's performance and used verbal rules to solve problems. Five additional kinds of knowledge were needed to make the model operational. As mentioned before, verbal rules tend to be high-level plans, so their action descriptions are stated in general terms. First, a semantic interpretation was needed that disambiguated and structured the descriptions into a set of actions that could be performed directly. For example, the disambiguation of the signed-number rule given above would have to include the fact that "Take the difference" refers to subtracting the absolute values of the numbers in the problem. Actions stated in the verbal rules were taken as goals. They were decomposed and given to the model as unambiguous descriptions of executable procedures. This elaborated representation of a verbal rule is similar to Sacerdoti's procedural net (Sacerdoti, 1977). Second, a set of procedures--which can be called parsing knowledge--were needed to determine the objects referred to in the verbal description. Objects extracted by these parsing procedures were needed by a third kind of knowledge--rule-selection procedures that determined whether a problem satisfied the application conditions of a verbal rule. Parsing procedures also provided arguments required by the fourth kind of knowledge--procedures that performed the executable actions in the elaborated rule representation. Fifth, executive control procedures

were needed to organize and direct performance. The operations of the last four kinds of knowledge correspond in a general way to the operations of a control schema in a pattern-directed inference system: selection, matching, execution, and scheduling (Waterman & Hayes-Roth, 1978, p. 11)

(2) Explicit recall and use of verbal rules in problem solving performance is optional and tends to disappear with practice. While performance sometimes includes the use of rules in a significant way, it often does not. With sufficient practice, cognitive skills apparently are organized in such a way that the mental procedures involved solely in explicit rule use can be omitted (e.g., Fitts, 1964; Neves & Anderson, 1981).

For example, on the first instance of a unlike-signs addition in the problem set, a student explicitly recalled and used a verbal rule. The problem was 30 + -29. The student's protocol was:

> OK. Thirty plus a negative 29. OK. Now what I do, the first thing I do is take the largest absolute value of these numbers which would be 30 and subtract the negative 29 and keep the sign of the larger absolute value. So the answer would be 1. It would be positive 1. Because the larger absolute value is positive.

On the sixth instance of an unlike-signs addition problem in the problem set, there was no evidence that verbal rules were used in a central way as in the first instance. One can also see that a number of explicit steps have dropped out of her performance. The problem was -35 + +39, and the protocol was: Negative 35 plus positive 39. That's subtracting, 4. That would be positive 4.

This principle is related to the next principle which describes another condition in which verbal rules tend to be used.

(3) Explicit recall and use of verbal rules tends to occur when directly applicable mental procedures are inadequate. Students are sometimes faced with problems to solve that they have not encountered recently. If a directly applicable mental procedure is not readily available for solution, then there is a tendency to attempt to retrieve and apply interpretatively an appropriate verbal rule. This principle is an instance of Ryle's (1946) claim that "people who appeal much to principles show they don't know how to act."

The protocol for 30 + -29 in the previous principle is an instance of this principle. This was the first occurence of a problem of this form, thus the student recalled a verbal rule as a plan for action. Explicit recalls were not limited to the early part of the problem set; some explicit recalls occurred even on the 25th problem of this set if it was a new form or such a form had not been solved recently. This result also shows that the second principle is not a consequence of the student becoming familiar with the interview procedure, thereby saying less as the interview proceeded.

The second and third principles show that verbal rules tend to be used by persons who are not practiced or competent at a skill. However, these principles also show that verbal rules can play an important role in the development of efficient mental procedures. Because verbal rules can be memorized and retrieved, a person is able to find a plan for problem solution if one is not immediately available. This adds a new

and useful point to Sacerdoti's (1977) planning formalism--a verbal rule provides a guide to action that is stored in a relatively accessible verbal form.

(4) Mental procedures derived from instructed verbal rules can have different structure than these rules and can be modified by prior knowledge. As we have shown already, a person must elaborate a verbal rule in order to make it operable psychologically. Unsuccessful attempts at elaborating verbal rules can result in incorrect procedures which if applied consistently can be called ill-composed. Ill-composed mental procedures can produce systematically flawed performance, similar to the manner described by Brown and Burton's (1978) BUGGY system in elementary arithmetic.

There are two results about ill-composed mental procedures that we want to report. They were derived by comparing the structure of the instructed verbal rules with the structure of the students's mental procedures. By this comparison we can show structural differences between the instructed and the acquired structures. To give the gist of the result, the instructed verbal rule given above for adding unlike-signs has a structure of "condition 1, condition 2; action 1, action 2". The entire rule is self-contained. On the other hand, one student uses two rules for the same kind of problem. The structure of both rules is "condition 1; action 1".

Second, other knowledge may be included in the procedures that are constructed from a verbal rule. For example, one student used a set of incorrect rules, identical to the sign assignment rules for signed

multiplication and division (SMD), to give the sign of addition and subtraction answers. The student learned the four signed operations at the same time. It appears that he overgeneralized the application of the SMD sign assignment operations and included it as part of his addition and subtraction procedures.

(5) Successful selection and execution of verbal rules requires parsing knowledge. When applying a verbal rule, one must be able to obtain information needed to select and correctly execute an appropriate rule. We will discuss three kinds of selection and execution difficulties that arise from having inadequate parsing procedures. First, there are times when a student must decide whether a particular problem falls into the domain of a set of rules that the student has available. This is primarily a problem of choosing a general approach for solving a problem, and it may be resolved by finding features in the problem that match the application conditions of a set of rules, or by asking an external source if the problem under consideration is an instance of a class of problems soluble by a particular method. For example, one student used an authority to decide that a set of rules should be used. Faced with "47 + 7", she asked the interviewer, "These are signed numbers, right?" When told they were, she went ahead and tried to recall how she was taught to solve signed-number problems and produced -54 as the answer. Second, if a student decides or knows that a problem can be solved by a particular set of rules, the student must then select a specific rule to use. Selecting a rule involves noticing the appropriate conditions that make that rule applicable to a particular problem. If these features are not noticed or noticed

incorrectly, then rules may be applied to problems for which they should not be used. For example a student misparsed the problem +46 - 12. He read it as "Positive 46, negative 12". He then went on to apply his rule for unlike-signs problems rather than the same signs rule, which he should have used. Third, a person will be unable to apply a selected rule, if that person is unable to extract the arguments needed for executing that rule. One student started immediately to apply a correct rule, but then hesitated because she was not sure that she had parsed the proper sign. The problem was 39 - -25 and her protocol was:

> "39 minus negative 25. You change the sign of the number being subtracted. So you change 25 to a positive and you change---. I just hope that I'm changing the right sign.

The preceding examples reinforce the idea that successful application of a verbal rule requires more than being able to recite it from memory. Parsing knowledge is needed to assist in the selection and execution of rules.

Conclusions

To summarize the main points of this paper, verbal rules provide a plan for action in a verbal-retrievable form, but students must add a non-trivial amount of knowledge to make the rules effective. Providing verbal rules as technique for solving problems does not guarantee that those rules will be used in a central way, nor that they will be learned the way they were taught. They may, however, be useful when a person's normal procedures are not available, or as a "training wheel" in the process of developing efficient mental procedures.

The analysis of the procedures constructed from the verbal rules may be a small step toward a theory of how verbal rules, or more generally, declarative statements are converted into procedures. We know what was taught, what the student may know before hand, and what knowledge the student has acquired. The obvious next step is to develop a theory of how the transitions occur.

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